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Random field modelling of DEM uncertainty and its impact on terrain referenced navigation

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Abstract

Terrain Referenced Navigation (TRN) is an integrated navigation solution, where terrain height measurements from a radar altimeter are compared to a Digital Elevation Model (DEM) to filter the errors of the Inertial Navigation System. Most conventional systems incorrectly assume that the DEM errors are Gaussian and uncorrelated, with a standard deviation linearly related to the slope. This is all the more annoying as any departure from these assumptions is known to adversely impact the TRN performance. In this paper, two new random field models of DEM altimetric error are described. The first model is a doubly stochastic random field. The error is Gaussian, conditioned on its standard deviation, modeled as a lognormal random field, whose mean is a logistic function of the DEM slope. This model is statistically learned by analyzing the difference of the DEM with a high-quality reference DEM. The second model is limited to Reference3D DEMs, with the availability of the (two) stereoscopic images used to produce the DEM. The approach relies on a Bayesian modelling of the altimetric error, where the prior is precisely the first model. The impact of the DEM uncertainty model on the TRN performance is evaluated through Monte Carlo simulation.

Keywords: DEM uncertainty, random field model, statistical learning, Bayesian stereo modelling, Terrain Referenced Navigation, uncertainty propagation.

1. Introduction

Digital Elevation Models (DEM) users are increasingly aware of the uncertainty associated with elevation data sets. Even when quality metadata are made available by the data provider, this information is generally not adequate to assess the risk of using the DEM in a given application (Goodchild *et al.*, 1999).

This situation is the more worrying that more and more high resolution DEM data, produced by aerial and satellite surveying, are made available on the market to support new, civil or military, applications.

IC3i has been funded for several years by the French MOD on modelling the uncertainty of Reference3D® level 2 DEMs (1 arc second) used in the French military forces (the DEMs are co-edited by IGN and Spot Image) and designing techniques to compute the impact of DEM uncertainty on Terrain Referenced Navigation (TRN) performance, cf. paragraphs 2 and 3.

In the nineties, geostatistical techniques (Fisher, 1998) have been introduced to model the spatial structure of the DEM error. Because the reference data were scarce, the techniques were essentially restricted to Gaussian (locally) stationary models, which are inadequate, if only because the standard deviation of the DEM error is known (Li *et al.*, 2005) to be roughly linearly related to the DEM slope, which is clearly non-stationary.

Nowadays, plenty of high quality reference data (Interferometric SAR or Laser DEMs obtained from aerial surveys) are available which allow for a detailed statistical modelling of operational DEM altimetric errors. Two techniques have been developed by IC3i, the first one being applicable to any DEM while the second one being specific to Reference3D®, when HRS stereoscopic data are made available (which is possible when buying the product to Spot Image).

The first model, described in paragraph 4, is a doubly stochastic random field. The error is Gaussian, conditioned on its standard deviation, modeled as a log-normal random field, whose mean is a logistic function of the DEM slope. Thus, the unconditional distribution of the error random field is non-Gaussian. This model is statistically learned from analyzing the difference of the DEM with an InSAR NEXTMap 40® DEM, edited by Intermap Technologies.

The second model, described in paragraph 5, is limited to Reference3D® DEMs, with the availability of the (two) stereoscopic images used to produce the DEM. It relies on a Bayesian modelling of the altimetric error, not unrelated to (Jalobeanu and Fitzenz, 2007). There are nevertheless important differences. In our approach, the DEM is given; the prior is our first model, parameterized by the DEM slope, and finally the probabilistic distribution of the stereo pair is based on a generalized Gaussian distribution, not merely a Gaussian distribution.

2. Principle of terrain referenced navigation

Terrain Referenced Navigation (TRN) is an integrated navigation solution, where terrain height measurements from a radar altimeter are compared to a DEM to filter the errors of the Inertial Navigation System (INS). The DEM can be either a Digital Terrain Model (DTM), which models the bare ground, or a Digital Surface Model (DSM), which models the Earth's surface and includes all objects on it. Most military TRN systems use level 1 DEM (grid spacing of 3 arc second).

Conventional TRN systems (Wilkinson *et al.*, 2009) determine terrain height by using a radar altimeter (radalt). The radalt has a large aperture to ensure the nadir point belongs to the main lobe of the radar antenna. In such a case the measured terrain height is nothing but the difference between the vehicle height (measured by the navigation system) and the radalt measurement.

Although this model is apparently used in most TRN systems, it is strictly valid only for flat terrains within the radar footprint. A more correct model is to assume that the radalt measures the minimum distance to the ground, or more precisely, to the nearest reflecting object on the ground. For that reason, it will be supposed in this paper that the DEM used for TRN is actually a DSM.

The integration filter (track-mode filter) is an extended Kalman filter, based on a linearization of the distance function to the DEM. Note that the radalt response is the minimum distance to the true terrain, which differs from the DEM by the altimetric error. The track-mode filter rejects radalt measurements when the innovation is three-times larger than its standard deviation computed by the Kalman filter.

The principle of TRN is schematized Figure 1.

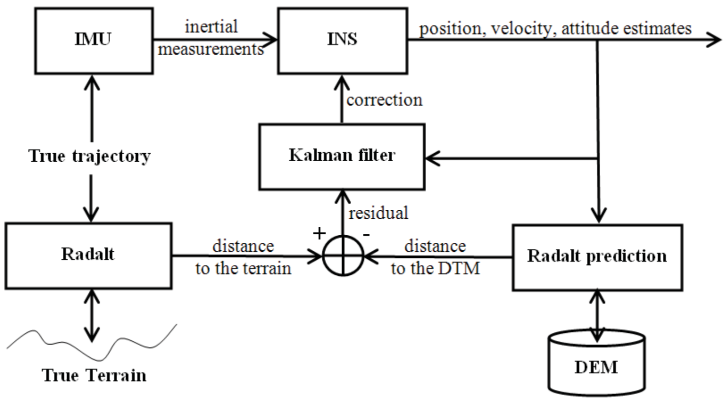

Figure 1: Principle of TRN.

The Kalman filter is fed by the difference of the distance to the terrain from the true position of the aerial vehicle and the distance to the DEM from the INS position. In the Kalman filter mechanization, this residual is approximated by:

$$\text{residual} \approx \frac{\partial \text{DEM}}{\partial X}\delta X + \frac{\partial \text{DEM}}{\partial Y}\delta Y + \delta Z + \varepsilon_{\text{RA}} + \varepsilon_{\text{DEM}} \quad (1)$$

(for simplicity, it is assumed that the radalt measures the vertical distance to the terrain), where (δX, δY, δZ) is the North, East and altitude INS error, ε_{RA} is the Radalt measurement error and ε_{DEM} is the DEM error.

3. TRN performance evaluation

TRN performance is assessed from both simulations and (expensive) real flights. In a simulation, a true trajectory (levelled flight or terrain following flight) of the aerial vehicle is given, from which are simulated the data from the Inertial Measurement Unit (IMU) and from the Radalt.

In the open literature, the true terrain is the DEM (preferably the DSM) to which is added a Gaussian white noise, whose standard deviation is a linear function of the DEM slope (in some cases, the true terrain is the DEM itself).

The TRN performance is usually assessed in terms of precision and robustness:

- The precision is evaluated by the 90% Circular Error Probable (CEP90) of the North and East (δX, δY) errors
- The robustness is evaluated by the probability that either |δX| or |δY| exceeds three times the corresponding standard deviation estimated by the Kalman filter.

The precision and robustness criteria are evaluated from Monte Carlo simulations, using the IMU, Radalt and DEM error models.

Note that, in a TRN performance evaluation by computer simulation, the planimetric error of the DEM can be neglected to a first approximation, as TRN is relative to the DEM. This is what will be assumed in the remainder of the paper.

It is well known in the aerospace industry that the simulations always give optimistic performance, compared to real flights. This can be explained by the heavy-tailed altimetric error distributions that will be presented in paragraphs 4 and 5.

4. The slope-based stochastic modelling of the altimetric error

The altimetric error, defined as the difference between the Reference3D® level 1 DEM and the (level 1-resampled) NEXTMap 40® reference DEM, satisfies:

$$\varepsilon_{DEM}(x,y) = \sigma(x,y)\xi(x,y) + b(x,y) \tag{2}$$

where b(x,y) is the bias field, assumed to be 0 for ease of exposition, and ξ(x,y) is a unit-variance Gaussian random field. In the literature (Li *et al.*, 2005), the standard deviation field σ(x,y) is assumed to be modeled by:

$$\sigma(x,y) = A + Bp(x,y) + w(x,y) \tag{3}$$

where p(x,y) is the DEM slope and w(x,y) is a Gaussian white noise.

This model provides a rather poor fit to the data, cf. Figure 3. IC3i has replaced (3) by a lognormal random field, whose mean is a logistic function of the slope:

$$\sigma(x,y) = S\big(p(x,y)\big)\exp\big(s\big(\eta(x,y) - s\big)\big) \quad S(x) = c_1 + c_2 / (1 + c_3\exp(-c_4 x)) \tag{4}$$

η(x,y) is a unit-variance zero-mean Gaussian random field. Note that for any s, $\sigma^2(x,y)$ is an unbiased estimator of $S^2(p(x,y))$. The parameter s>0 is used to fit the histogram of ε_{DTM}. The quality of the sigmoid fit is shown Figure 2.

The modelling is complete, once we have defined the covariance function of the Gaussian fields ξ(x,y) and η(x,y). It is clear that the fields cannot be white noises, if only because, for cosmetics reasons, all data producers smooth DEMs. IC3i has found that an acceptable trade-off between computational simplicity and accuracy modelling is to assume that ξ(x,y) and η(x,y) are two independent stationary Gaussian random fields with the same powered-exponential correlation function:

$$\rho(x,y) = E[\xi(x+u, y+v)\xi(u,v)] = \exp\left(-3\left(\frac{(dLon * x)^2 + (dLat * y)^2}{R^2}\right)^H\right) \tag{5}$$

where (dLon, dLat) denote the (level 1) DEM pixel sizes (in m) in longitude and latitude; R denotes the correlation length of the random fields ξ(x,y) and η(x,y); H is the Hurst coefficient related to the regularity of the fields. The parameters R and H are estimated from the stationary autocorrelation of the field $\varepsilon_{DEM}(x,y)$:

$$c(x,y) = \frac{1}{N^2}\sum_{i=1}^{i=N}\sum_{j=1}^{j=N}\varepsilon_{DEM}(x+i, y+j)\,\varepsilon_{DEM}(i,j) \tag{6}$$

which is a function of R, H and the stationary autocorrelation of the field p(x,y).

The realizations of $\varepsilon_{DEM}(x,y)$ are efficiently computed by FFT using the circulant embedding technique described in (Rue and Held, 2005), paragraph 2.6.4.

The results of the modelling are shown Figure 5.

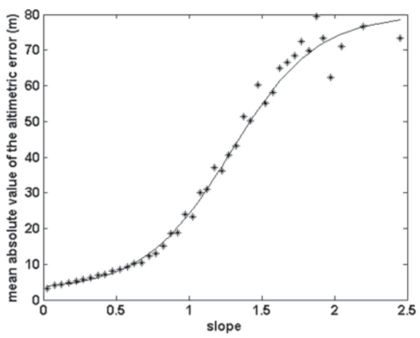

Figure 2: mean absolute value of ε_{DTM} as a function of the DEM slope

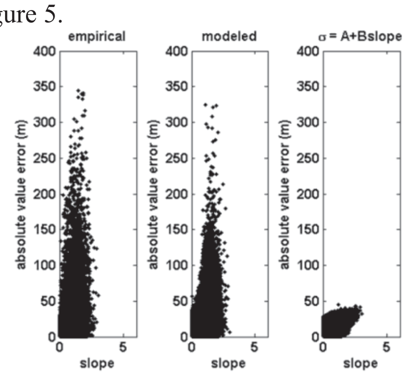

Figure 3: Scatterplot of the absolute value of ε_{DEM} versus the DEM slope

5. The Bayesian stereo modelling model of the altimetric error

The operational DEMs (DSMs) used in the French Armed Forces are produced by (semi) automatic correlation of SPOT 5 along-track stereo images. It is well known that the performance of automatic correlation is adversely affected by the slope of the terrain, but bad correlation may also be caused by departure from the Lambertian reflection model. The second DEM uncertainty model will be precisely derived from the analysis of the local correlability of the stereo images.

Recently, there has been a marked interest in the literature in applying Bayesian methodology to stereo problems (Szeliski, 1989). One of the recurring problems in applying Bayesian methodology is the prior distribution modelling. The slope-based stochastic modelling of the altimetric error, described in paragraph 4, provides such a prior model. This implies that the natural framework for the stereo matching is the object space and not the epipolar image space (Kasser and Egels, 2001).

Figure 4 depicts the functional chain of the IC3i technique. The stereopair $\{I_1,I_2\}$, supposed to be available with the DEM, is used to compute pairs of ortho-images $\{I_1(\Delta Z),I_2(\Delta Z)\}$, when the true DEM is equal to the sum of the given DEM and the error field $\Delta Z(x,y)$. If the DEM is correct, the stereopair $\{I_1(\Delta Z),I_2(\Delta Z)\}$ should have the highest (local) correlation when $\Delta Z(x,y)=0$.

The likelihood $L(I_1(\Delta Z),I_2(\Delta Z))$ is built from a local similarity measure. Most measures of the open literature (Jalobeanu and Fitzenz, 2007) use the square of $I_1(\Delta Z)-gI_2(\Delta Z)-b$, where g and b are locally constant. This amounts to assuming that the distribution of $I_1(\Delta Z)$, given $I_2(\Delta Z)$, is Gaussian. IC3i has discovered that superior results are obtained by assuming a generalized Gaussian distribution.

Using the results of (Rue and Held, 2005), it is possible to approximate the prior probability of $\xi(x,y)$ by a Gauss-Markov Random Field (GMRF) with a (5x5) neighbourhood. Thus, we simulate the altimetric error as follows:

- firstly, we simulate a prior standard deviation field σ from equation (4),
- secondly, we compute the Laplace approximation (Rue and Held, 2005, p. 212) of the posterior distribution of ε_{DEM} field given I_1, I_2 and σ.
- thirdly, we simulate a realization of $\varepsilon_{DEM}(x,y)$ from the (approximate) Gaussian distribution of ε_{DEM} given σ, using the techniques of GMRF simulation of (Rue and Held, 2005).

Note that the unconditional distribution of $\varepsilon_{DEM}(x,y)$ is non-Gaussian, because of the prior on $\sigma(x,y)$. Although this approach leads to superior modelling results compared to the first method, it requires a much higher computational power.

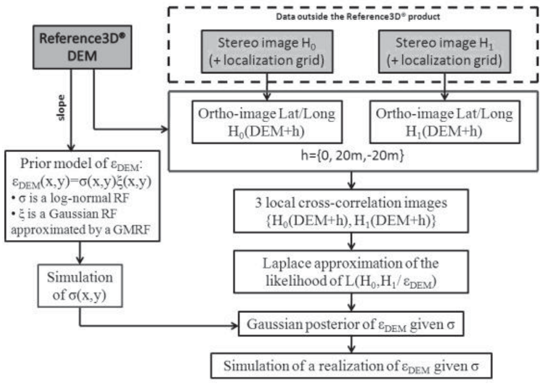

Figure 4: Functional chain of the altimetric error Bayesian stereo modelling

6. Results

The modelling results of the two above techniques are presented Figure 5. The left image is the E005-N45 Reference3D® DEM. The middle and right images, displayed with a logarithmic scale, correspond to realizations of the standard deviation random field for the 1st and 2nd methods (the non-zero part of the right image corresponds to the footprint of a SPOT 5 HRS segment). Figure 5 clearly shows that the altimetric error is "correlated" with the morphometry of the terrain. Also note that the right image is more textured than the middle one.

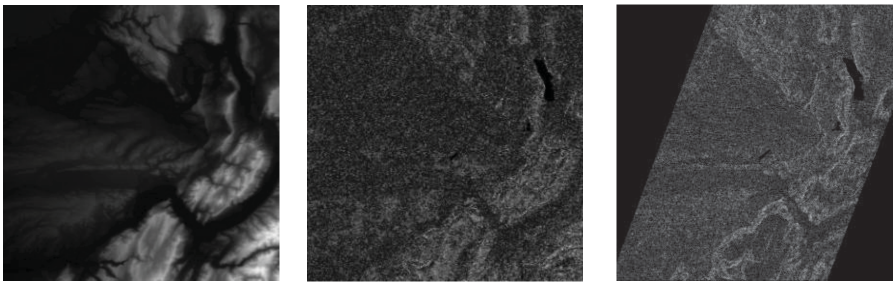

Figure 5: The DEM Reference3D® and realizations of σ(x,y) [1st, 2nd method]

The Monte Carlo simulations have shown, for different trajectories chosen within the E005-N45 DEM, that the CEP90 achieved by a TRN system is roughly twice the value obtained when the true terrain is taken to be the DEM itself. The probability of the error exceeding the 3σ uncertainty is also dramatically increased.

To summarize, this paper has shown that DEM errors are non-Gaussian, non-stationary and autocorrelated. Neglecting these can lead to serious performance degradations for operational applications, as it was demonstrated for TRN.

References

Fisher, P. (1998). "Improved Modelling of Elevation Error with Geostatistics", GeoInformatica, Vol. 2(3):215-233.

Goodchild, M.F., Shortridge, A. , Fohl, P. (1999). "Encapsulating simulation models with geospatial data sets". *In:* Lowell, K. and Jaton, A. (eds.). *Spatial Accuracy Assessment: Land Information Uncertainty in Natural Resources*, Ann Arbor Press, pp. 123-130.

Heuvelink, G. B. M. (2000). *Error Propagation in Environmental Modelling with GIS*, Taylor & Francis, 127 p.

Jalobeanu, A. and Fitzenz, D. (2007). "Robust disparity maps with uncertainties for 3D surface reconstruction or ground motion inference". *Proceedings of the ISPRS Workshop on Photogrammetric Image Analysis (PIA'07)*, Munich, September 2007.

Kasser, M. and Egels, Y. (2001). *Digital Photogrammetry*, Taylor and Francis, 351 p.

Li, Z., Zhu, Q. and Gold, C. (2005), *Digital Terrain Modelling*, CRC Press, 323 p.

Rue, H., Held, L. (2005), *Gaussian Markov Random Fields*, Chapman & Hall/CRC, 263 p.

Szeliski, R. (1989). *Bayesian Modelling of Uncertainty in Low-level Vision*, Kluwer Academic Publishers, 198 p.

Wilkinson, N., Brooks, T., Price, A. (2009). "Latest Developments of the TERPROM® Digital Terrain System 2009". *Joint Navigation Conference*, Orlando, Florida.